

A New Framework for Multijet Predictions and its application to Higgs Boson production at the LHC

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We present a new framework for calculating multi-jet observables through resummation. The framework is based on the factorisation of scattering amplitudes in an asymptotic limit. By imposing simple constraints on the analytic behaviour of the result when applied away from this limit, we get good agreement with the known lowest order perturbative behaviour of the scattering amplitude, and predictions for the behaviour to all orders in the perturbative series. As an example of application we study predictions for Higgs Boson production through gluon fusion at the LHC in association with at least two jets.

Events with many energetic particles will form the backbone of many search strategies at the LHC for physics beyond the Standard Model (SM) of particle physics, but will simultaneously test our understanding of SM processes. Energetic particles of QCD charge will be detected as jets, and events with multiple jets require the calculation of scattering amplitudes with a high number of external quark and gluon (parton) legs. However, the calculation of the SM contribution to such processes beyond even the lowest order in perturbation theory is notoriously difficult. Despite recent impressive progress in the calculation of many multi-leg scattering amplitudes at both tree and one-loop level, the first two radiative corrections in the perturbative series (i.e. full next-to-next-to-leading accuracy) for observables at the LHC are known only for very few cases[1, 2, 3].

One estimate of the effect of higher order corrections beyond what is currently calculable in full fixed-order perturbation theory can be obtained by interfacing the fixed perturbative calculation with *parton shower* programs[4, 5], which sum the effect of further *soft and collinear* radiation (i.e. of low invariant mass) to all orders in perturbation theory.

As we will demonstrate in this paper, there are important processes where the perturbative corrections from *hard* radiation (i.e. of large invariant mass) is sizeable. In such cases, the parton shower approach cannot be expected to capture the dominant effect of higher order corrections. In this letter we will develop an alternative formalism for estimating the all-order perturbative corrections, which does not rely on the description of only soft and collinear radiation. Instead, the formalism will utilise results on the factorisation of matrix elements in the opposite limit of infinite invariant mass between all radiated particles. We will see that without further modifications, the results obtained in this limit lead to a poor approximation of LHC scattering amplitudes. However, known analytic properties of the full perturbative amplitude can be incorporated, such that the approximate amplitudes agree well with the full result at low orders in perturbation theory. The approximate amplitudes can

then be applied to any order in the perturbation expansion, where the full results are incalculable.

We will validate the approach by considering Higgs boson production through gluon-gluon fusion (GGF) (mediated by a top-quark loop) in association with at least two jets (the hjj -channel). We will explicitly compare our approximate amplitudes with those obtained using full fixed order perturbation theory, where such results are known. This process can be used to search for the Higgs boson [6], and also potentially to measure its couplings to the top quark [7]. The hjj -channel also has a large contribution from *weak boson fusion* (WBF)[8, 9, 10], and by suppressing the GGF-contribution it is possible to measure the coupling of any Higgs boson candidate to the electro-weak bosons, and thus determine[11] whether the properties of the candidate match those of the SM Higgs boson. The suppression of the GGF-contribution is achieved[12] to some degree by applying specific event selection criteria, as discussed in Table I. However, only by calculating higher order corrections is it possible to estimate the efficiency of such cuts.

The current state of the art in fixed order perturbation theory for Higgs boson production through GGF in association with at least two jets includes only the first perturbative corrections, and is presented in Ref. [13]. The effects of further soft and collinear radiation were studied in Ref. [14]. In the limit of infinite top mass, the coupling of the Higgs boson to gluons through a top quark loop can be described as a point interaction[15, 16, 17]. This approximation was applied in all these studies, and will be applied also in the present one, although this is not essential to the approach.

We will start motivating the need for considering the hard radiative corrections to higher orders by studying the leading order predictions in full QCD for Higgs boson production in association with hard jets. We apply similar event selection cuts to the study of Ref.[13] as detailed in table I[29]. All of the following results are obtained by choosing renormalisation and factorisation scales in accordance with the study of Ref.[14], and the following values for the Higgs boson mass, vacuum ex-

$p_{j\perp} > 40 \text{ GeV}$	$y_c \cdot y_d < 0$
$ y_{j,h} < 4.5$	$ y_c - y_d > 4.2$

TABLE I: The cuts used for all results in this letter, in terms of rapidities y_i and transverse momenta $p_{i,\perp}$. The suffices c, d label cuts that must be satisfied by at least two jets, whereas j labels conditions that must be satisfied by all jets; h labels the Higgs boson.

pectation value of the Higgs boson field and top quark mass respectively:

$$m_H = 120 \text{ GeV}, \langle \phi \rangle_0 = \frac{v}{\sqrt{2}}, v = 246 \text{ GeV}, m_t = 174 \text{ GeV}.$$

We also include a factor multiplying the effective Higgs Boson vertices, accounting for finite top-mass effects [18]:

$$K(\tau) = 1 + \frac{7\tau}{30} + \frac{2\tau^2}{21} + \frac{26\tau^3}{525}, \quad \tau = \frac{m_H^2}{4m_t^2}.$$

We choose the k_t -jet algorithm as implemented in Ref.[19] with $R = 0.6$, and the parton distribution functions of Ref. [20]. For the strong coupling α_s , we choose renormalisation scales as in Ref. [14] ([30]). With this, we find (using matrix elements from MADEvent/MADGraph[21]) the tree-level cross-section from the QCD generated hjj -channel to be $281_{-111}^{+210} \text{ fb}$, where the uncertainty is obtained by varying the common factorisation and renormalisation scale by a factor of two. For the three jet sample, all jets must satisfy the left-most cuts of Table I, but we also require that there exist two jets c, d satisfying all the right-most cuts in Table I. We then find the leading order cross section for the production of Higgs Boson plus three jets ($hjjj$) to be $257_{-120}^{+262} \text{ fb}$. The requirement of an extra hard jet with an accompanying α_s -suppression leads only to a 9% suppression compared to the leading order prediction for hjj . The α_s -suppression of the matrix element is compensated by the integration over a large phase space for the third jet. The large size of the three-jet rate (which obviously depends on the event selection cuts) was already reported in Ref.[22], and clearly demonstrates the necessity of considering hard multi-parton emissions.

We will now describe a method for approximating the perturbative scattering matrix elements for multi-particle production to any order. We start by recalling the result of Fadin and collaborators (FKL)[23], that in the limit of infinite invariant mass between all produced particles (the Multi-Regge-Kinematic (MRK) limit), the leading contribution is given by processes of the form

$$\alpha(p_a) + \beta(p_b) \rightarrow \alpha(p_0) + \sum_i^{n-1} g(p_i) + \beta(p_n) + h(p_h)$$

where $\alpha, \beta \in \{q, \bar{q}, g\}$, and the partons are ordered according to rapidity in both the initial and final state. These processes allow neighbouring particles to be connected by gluon propagators of momentum q_i , such that

$p_i = q_i - q_{i+1}$. We have explicitly checked that at leading order, partonic configurations which are not captured in this framework account for only 0.8fb (0.3%) and 24fb (< 10%) of the two and three-jet rate respectively, even when there is no requirement of large invariant mass between all particles.

In the MRK limit, the scattering amplitude for the remaining configurations factorises, and the results of Ref.[23] can straightforwardly be modified to include also the production of a Higgs boson. In the case of the Higgs boson being produced between the jets (in rapidity) these amplitudes take the form:

$$\begin{aligned} i\mathcal{M}_{\text{FKL}}^{p_a p_b \rightarrow p_0 \dots p_j h p_{j+1} p_n} &= 2i\hat{s} \left(i g_s f^{a d_0 c_1} g_{\mu_a \mu_0} \right) \\ &\cdot \prod_{i=1}^j \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i)(y_{i-1} - y_i)] \left(i g_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \\ &\cdot \left(\frac{1}{q_h^2} \exp[\hat{\alpha}(q_i)(y_j - y_h)] C_H(q_{j+1}, q_h) \right) \\ &\cdot \prod_{i=j+1}^n \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i)(y'_{i-1} - y'_i)] \left(i g_s f^{c_i d_i c_{i+1}} \right) C_{\mu_i}(q_i, q_{i+1}) \right) \\ &\cdot \frac{1}{q_{n+1}^2} \exp[\hat{\alpha}(q_{n+1})(y'_n - y_b)] \left(i g_s f^{b d_{n+1} c_{n+1}} g_{\mu_b \mu_{n+1}} \right), \end{aligned} \quad (1)$$

where g_s is the strong coupling constant ($\alpha_s = \frac{g_s^2}{4\pi}$); f^{abc} colour structure constants; y_i, y'_i are the rapidities of the emitted particles; $\hat{s} = (p_a + p_b)^2$ is the partonic centre of mass energy. The factor C_{μ_i} is a *Lipatov effective vertex* describing the emission of gluon i . This has the explicit form:

$$C^{\mu_i}(q_i, q_{i+1}) = \left[(q_i + q_{i+1})_{\perp}^{\mu_i} - \left(\frac{\hat{s}_{ai}}{\hat{s}} + 2 \frac{q_{i+1}^2}{\hat{s}_{bi}} \right) p_b^{\mu_i} + \left(\frac{\hat{s}_{bi}}{\hat{s}} + 2 \frac{q_i^2}{\hat{s}_{ai}} \right) p_a^{\mu_i} \right], \quad (2)$$

where $\hat{s}_{ai} = 2p_a \cdot p_i$ and similarly for \hat{s}_{bi} . The notation q_{\perp} denotes the projection of a 4-momentum onto its transverse components. Also, C_H is an effective vertex coupling the Higgs to off-shell gluons via a top quark loop, whose form has been calculated in [24]. The exponential factors $\hat{\alpha}(q_i)$ encode the leading virtual corrections (see e.g. Ref [25]):

$$\hat{\alpha}(q_i) = -\frac{g_s^2 N_c \Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon} (|q_{\perp i}|^2 / \mu^2)^{\varepsilon}, \quad (3)$$

where singularities have been regularised using dimensional regularisation in $D = 4 + 2\varepsilon$ dimensions, where μ is the renormalisation scale and N_c the number of colours. The colour factors in equation (1) are for incoming gluons. The form of the amplitude is the same for initial state quarks, apart from different colour factors.

Eq. (1) formally applies in the so-called *Multi-Regge-kinematic* (MRK) limit. Thus, it is clear why this may be a good starting point for describing matrix elements with many hard partons. Firstly, it can be applied at any order in the perturbation expansion. Secondly, it does not rely upon soft and collinear approximations.

The multi-gluon emissions have in fact not previously been studied directly as implemented in Eq. (1). Instead, a simplified version has been used, which is equivalent in the MRK limit. In this limit, the squared 4-momenta fulfil $q_i^2 \rightarrow -|q_{\perp i}|^2$, and the squared Lipatov vertices $-C_{\mu_i} C^{\mu_i} \rightarrow 4 \frac{|q_{\perp i}|^2 |q_{\perp i+1}|^2}{|k_{\perp i}|^2}$. This means that in the products of Eq. (1), only squares of *transverse* momenta appear. Extending these kinematic approximations to all of phase space (not just the MRK limit), the sum over j, n and the phase space integral over the emitted gluons can be approximated by solving the *BFKL-equation*[26]. In this form, the framework has previously been extensively applied to other processes. In the present context (after implementing local 4-momentum conservation, which is strictly sub-leading in the BFKL approach), we find by expanding the solution in powers of α_s that the lowest order BFKL results for the hjj (554fb) and $hjjj$ (775fb) cross sections differ from their full leading order counterparts by 97% and 200% respectively. The kinematic approximations are clearly inadequate in describing amplitudes in general at the LHC.

Instead, we define a set of amplitudes based on Eq. (1) as written, supplemented by the following guidelines:

1. Use of full virtual 4-momenta: Rather than substituting $q_i^2 \rightarrow -|q_{\perp i}|^2$ as in the BFKL equation, we keep the dependence on the full 4-momenta of all particles. This ensures that outside of the MRK limit, the singularity structure of the approximate amplitudes coincides with known singularities of the full fixed order scattering amplitude.
2. Positivity of the squared Lipatov vertex: The square of the amplitudes in Eq. (1) are not positive definite, when the effective Lipatov vertex is applied to momentum configurations very far from the MRK limit. It is here possible to obtain $-C_{\mu_i} C^{\mu_i} < 0$, where the minus sign arises from the contraction of the gluon polarisation tensor. We choose to remove the contribution from the *small* region of phase space where this happens.

These modifications combine known analytic behaviour of the full scattering amplitudes and the factorised expressions obtained in the MRK limit to any order in perturbation theory.

When these modifications are made, the resulting amplitudes do indeed approximate well the known perturbative results at low orders, and thus can be reliably used at higher orders, where full results are unknown or computationally unfeasible. Using our approach, we find an α_s^4 contribution to hjj of 321fb and α_s^5 -contribution to $hjjj$ of 217fb, within 16% and 7% of the full results respectively. In general, we find good agreement in a large region of phase space, and the level of accuracy reported here does not require any fine tuning of cuts. This is summarised in Fig. 1, and one sees that the approximate results are well within the uncertainty associated with

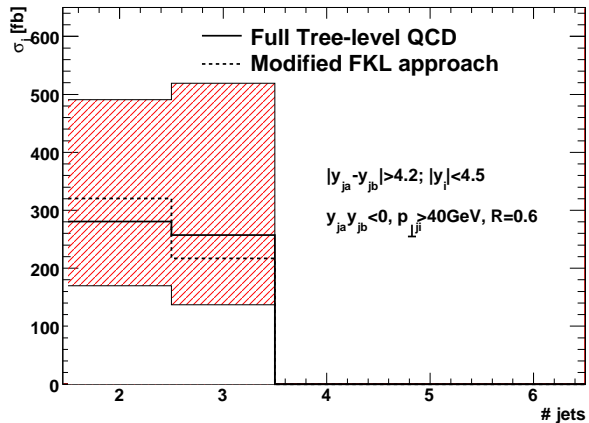


FIG. 1: The 2 and 3 parton cross-sections calculated using the known LO matrix elements (solid), and the estimate gained from the modified high energy limit (dashed). The uncertainty band on the LO result corresponds to scale variation by a factor of two.

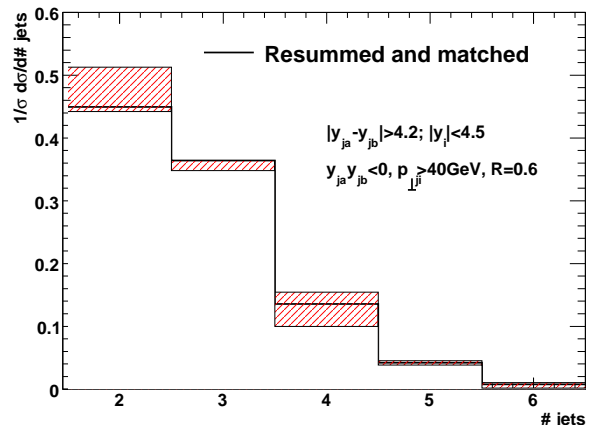


FIG. 2: Distribution of number of jets, obtained using the modified FKL approach, matched to the exact tree level results for 2 and 3 partons. The shaded uncertainty bands are obtained by scale variations.

the full results, obtained by varying a common renormalisation and factorisation scale by a factor of two.

Having validated the approximation at low orders in α_s (where it can be compared with known fixed order results), we now consider results obtained using matrix elements with any number of final state partons, which at present cannot be calculated using standard perturbation theory. The divergence in Eq. (1) arising when any $p_i \rightarrow 0$ is regulated by the divergence of the virtual corrections encoded in $\hat{\alpha}$. Thus the resulting formalism is efficiently implemented in a Monte Carlo generator following the method for phase space generation outlined in

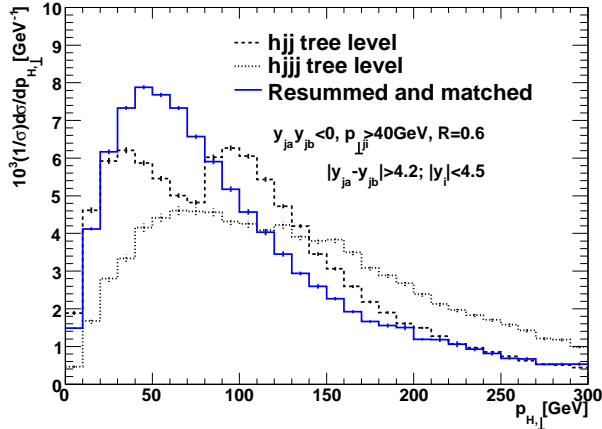


FIG. 3: The transverse momentum spectrum of the Higgs boson in the process $pp \rightarrow h + n\text{jets}$ at tree-level for $n = 2$, $n = 3$, and the transverse momentum spectrum obtained in the resummed approach presented in this paper.

Ref. [27]. Given that one knows the full tree level results for 2 and 3 partons, however (and they are also computationally quick), we have combined these results with the approximate matrix elements using a suitable matching procedure. We find a total cross section of $499^{+527}_{-307}\text{fb}$. The large uncertainty in the total cross section due to scale variations however is not reflected in the distribution of the number of hard jets as shown in Fig. 2. One sees a significant number of events with more than 3 hard jets.

The transverse momentum spectrum of the Higgs boson when produced in association with at least two hard jets is shown in Fig. 3. To the best of our knowledge,

this is the first report of this quantity, in contrast to the completely inclusive Higgs boson p_T spectrum, which has previously been studied in the literature. The tree level 2 parton final state predicts a bimodal structure, which ultimately arises from the azimuthal correlation between the jets. This structure disappears when extra radiation is added, giving a qualitatively different behaviour. The significant difference between the fixed-order spectra emphasises the importance of considering yet higher order corrections.

In summary, we have outlined a technique, not relying on a soft and collinear approximation, for estimating scattering amplitudes with multiple partons to all orders in perturbation theory, and demonstrated its application to Higgs boson production (via GGF) in association with at least two jets. Our technique is based on the FKL factorisation formula of Ref. [23], with important modifications which ensure that the singularity structure of the amplitudes coincides with known all-order analytic properties of the perturbation expansion. At low orders in α_s , where the full fixed order result can be obtained, our description agrees well, which verifies the trustworthiness of the approach. It captures both real and virtual corrections, and can be applied at any order in α_s in a computationally efficient manner.

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- [29] If further a rapidity veto on jets is applied, the effects studied in Ref. [28] may need to be taken into account.
- [30] For the resummed results presented later, some freezing of the coupling α_s is necessary below a suitable low scale Q_0 . However, the results are fairly insensitive to this choice.